Freshman Seminar 23p

Physics, Math and Puzzles

Cumrun Vafa

(W 4:30-6:30, Jefferson-453)

Physics is a highly developed branch of science with a broad range of applications. Despite the complexity of the universe the fundamental laws of physics are rather simple, if viewed properly. This seminar will focus on intuitive as well as mathematical underpinnings of some of the fundamental laws of nature, as well as how to appreciate the beauty of the laws. The seminars will use mathematical puzzles to introduce the basic features of physical laws. Main aspects discussed include the role of symmetries and their breaking in physical theories, the role of intuition as well as the power of modern math, including abstract ideas in topology, in unraveling the mysteries of the universe. The issue of why the universe is so big, as well as its potential explanation is also discussed. This seminar is recommended for students with a strong background in both math and physics and with keen interest in the relation between the two subjects.

Course Syllabus

Assignments

Students are expected to complete weekly readings and solve all the puzzlesand questions specifically introduced for the weekly discussions. Students are also expected to participate in class discussions. At the end of the seminar, each student will choose one of the weekly topics and prepare an in-class presentation on it. A satisfactory grade will be given to students completing all of the readings and weekly assignments as well as the final presentation on week 11.

References

References for this course include readings from books written for general public as well as additional sources. In particular use will be made of Richard Feynman's Cornell Video Lectures, Brian Greene's book, The Elegant Universe (1999) and Leonard Susskind, The Cosmic Landscape: String Theory and the Illusion of Intelligent Design (2006).

Academic Integrity:

Collaboration on assignment is permitted, but you should write and submit your own understanding of the solutions to puzzles and questions.

Weekly Sessions:

Week 1- General Introduction to Modern Physics

Assignment 1

1. Watch the second and third lectures of Feynman in
http://www.youtube.com/watch?v=zQ6o1cDxV7o&list=PL85FE0A635887998C

The second lecture is on the relation between physics and mathematics and the third lecture is on conservation laws.

2. Puzzle: Chess domino.

Week 2- Symmetry and Conservation Laws

Assignment 2

1. Watch the fourth Feynman lecture, `The Character of Physical Law: Symmetry in Physical Law'.
http://www.youtube.com/watch?v=8K09ylkIVNQ

2. Puzzles: Time reversal symmetry, deck ordering, milk/water, square grid graph drawing

3. Readings: Excerpts from Brian Greene's Book.

Week 3- Symmetry Breaking

Assignment 3

1.Gravitational Lensing: Watch the youtube video:
http://www.youtube.com/watch?v=yamVbK-J69M

2. For a more extensive discussion of the physics of gravitational lensing look at the article below. Do not attempt to read the whole paper, just get a flavor of what kind of physics is involved.

http://relativity.livingreviews.org/Articles/lrr-1998-12/download/lrr-1998-12Color.pdf

For some additional images of gravitational lensing you can see:
http://csep10.phys.utk.edu/astr162/lect/galaxies/lensing.html

3. Puzzles: Equator temperature, minimal road construction on a square.

Week 4- Power of Simple and Abstract Math, including: images of galaxies due to bending of light, temperature on equator, temperature and pressure on surface of earth

Assignment 4

1. Puzzles: Chocolate breaking puzzles, tennis tournament.
2. Readings: excerpts from Susskind's Book.

Week 5- Counter-Intuitive Math, including: modern examples arising from string theory

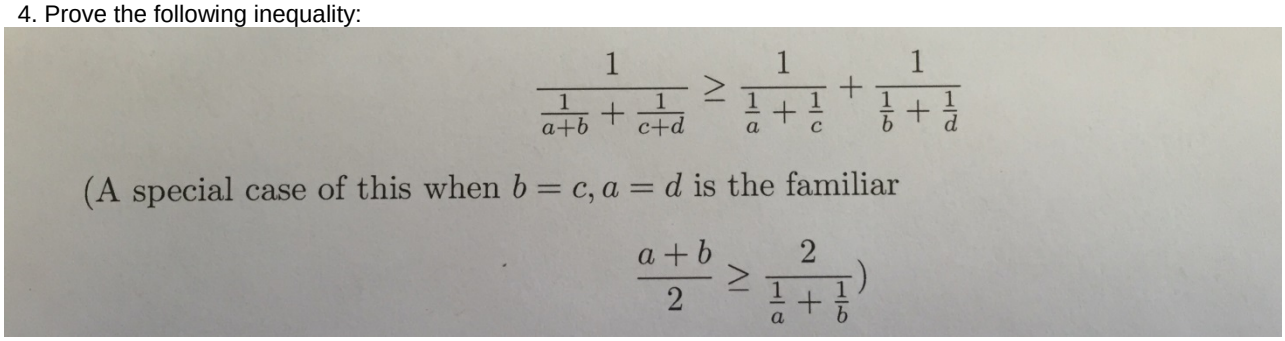
Assignment 5

1. Read Stephen Wolfram's account of Feynman's way of building physical intuition: http://www.stephenwolfram.com/publications/recent/feynman/

2. Puzzles: Mirror symmetry, holography, geometric singularities

3. Excerpt from Feynman's lecture series.

4. Prove the following inequality:



$$\frac{1}{\frac{1}{a+b}+\frac{1}{c+d}} \geq \frac{1}{\frac{1}{a}+\frac{1}{c}}+\frac{1}{\frac{1}{b}+\frac{1}{d}}$$

(A special case of this when $b = c, a = d$ is the familiar

$$\frac{a+b}{2} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}})$$

Try proving it mathematically! Alternatively you can use a resistor network to prove the inequality using physics intuition: Construct a network for which the left term gives the resistance of the network and use the fact that resistance of the network will go down if one adds a resistor whose removing leads to the same network.

Week 6- Physical intuition, including: how Newton and Galileo thought as well as buoyancy

1. Watch Feynman's Lecture on \Probability and Uncertainty: The Quantum Mechanical View of Nature": http://www.youtube.com/watch?v=aAgcqgDc-YM

2. Puzzle: Beach ball.

3. Can you make a time machine (at least theoretically)? The idea is to make the design so that you can travel at least in one direction in time. Which direction of time travel is possible, and how do you go about such a design? Provide the basic specifications of your design if the time travel is to take you in a 1000 year time journey and you would like the journey to be just long enough for you to watch a 2 hour movie!

3. Readings:
   Excerpts from Brian Green's Book.
   Einstein-Podolsky-Rosen (EPR) paradox: https://en.wikipedia.org/wiki/EPR_paradox

Week 7- Counter-Intuitive Physics: Relativity (twin paradox) and Quantum Mechanics (light passing through objects).

Assignment 7

1. Why does a matrix satisfy its own characteristic equation? (Hint: This is partly a joke!)

2. An episode of Mythbusters involved trying to duplicate the bus jump stunt in the movie `Speed'. They made a 1:12 scale model of the bus and bridge. But how should they scale the 50 mph speed of the full-size bus? (Do not use any equations from mechanics {just use the fact that it should depend only on g; v;L, the gravitational constant, speed of the bus and length of the bridge.)

3. Suppose we scale up human beings by a factor of 100, without changing the material we are made of. Would there be any problems?

4. Compute the area of the horizon of a black hole of mass M up to an overall numerical constant. Using this, estimate the radius to which our sun needs to be shrunk in order for it to become a black hole. You are not to use any equations of general relativity! (Hint: In Einstein's theory of relativity, the only fundamental parameters that appear are G, the Newton's constant, and c the speed of light).

5. In the above problem you found the estimate for the radius R of a black hole as a function of mass. Estimate the entropy of a black hole (which is dimensionless) assuming that it is a quantum effect (i.e. Planck's constant ~~h~~ which has dimensions of energy-time) and that it is proportional to its area.

6. A particle of mass m is in a box of size L. In classical mechanics the minimal energy this can have is 0. But in quantum theory it is not 0. Explain why it is zero in classical theory and why it can be non-zero in quantum theory without using any equations from classical mechanics or quantum mechanics (you can only use the fact that _h appears in quantum theory but not in classical theory).

Read: Victor Weisskopf's "Modern physics from an elementary point of view"
http://cds.cern.ch/record/274976/_les/CERN-70-08.pdf

Week 8- Naturalness in Physics, including: power of dimensional analysis

Assignment 8

1. Readings:
   Archimedes' Cattle problem
   https://www.cs.drexel.edu/~crorres/Archimedes/Cattle/cattle vardi.pdf

   Sean Carroll's review `The Cosmological Constant':

http://ned.ipac.caltech.edu/level5/Carroll2/frames.html

The Hierarchy Problem and the Anthropic Principle:
http://en.wikipedia.org/wiki/Hierarchy problem
http://en.wikipedia.org/wiki/Anthropic principle

R. Buosso and J. Polchinski on string theory Landscape:
http://www.pha.jhu.edu/courses/172 113/BoussoPolchinski.pdf

2. Puzzle: Why a matrix satisfies its own characteristic equation?

3. Check the identity: $95800^4 + 217519^4 + 414560^4 = 422481^4$:
Can you find any smaller positive integers such that the sum of three fourth powers is another fourth power?

Week 9- Unnaturalness and Large Numbers: Counter Example to Euler's Conjecture, Cosmological Constant, Anthropic Principle and String Landscape.

Assignment 9 and 10

1. Readings:

Newton's view on religion: *ttp*: *//en.wikipedia.org/wiki/Isaac Newton*'*s religious views*

Einstein's views on religion: *http*: *//en.wikipedia.org/wiki/Religious views of Albert Einstein*

Georges Lemaitre and his founding of the Big Bang Theory: *http*: *//en.wikipedia.org/wiki/Georges Lemaitre*

The relationship between various religions and science:*http*: *//en.wikipedia.org/wiki/Relationship between religion and science*

2. Watch:

Feynman's take on religion:*http*: *//www.youtube.com/watch?v* = 3*zi*699*WzAL*0

Brian Greene's Nova video: http://www.pbs.org/wgbh/nova/physics/fabric-of-cosmos.html#fabric-space

3. Puzzle: Dominos on square grids.

Week 10- Physics and Religion – Cont'd (where there is smoke, there is fire, or is there?)

Week 11- Power of Duality

1. Readings: Witten's article 'Reflections on the Fate of Spacetime' *http* : *//www.sns.ias.edu/* □ *witten/papers/Reflections.pdf*

2. Puzzle: Ant colliding on a stick.

3. Prepare for your in-class presentation.

Week 12- Summary & Lessons Learned.